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Indicial Approach to Harmonic Perturbations in Transonic Flow

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I. Introduction

BALLHAUS and Goorjian[1] suggest that harmonic aerodynamic force coefficients in transonic flow be calculated from the time-dependent loading generated following a step change in angle of attack. Their upwash involves a step change with time, and this step change has to be modeled in a finite difference approximation to the equations of motion. The step change in upwash is regarded as causing a perturbation to the steady transonic flow and, although the governing equations of the flow are nonlinear, their procedure for obtaining the harmonic force coefficients is strictly valid only if the perturbation constituent of the flow may be regarded as being governed by linearized equations. Rizzetta and Chin[2] also determine the loading generated following a step change in angle of attack and, in contrast to Ballhaus and Goorjian,[1] they do get an abrupt change with time in the loading. Ballhaus and Goorjian[1] make a low-frequency approximation in the governing differential equation of transonic flow, whereas Rizzetta and Chin[2] do not.

The step change in upwash is rather severe for a finite difference approximation to cope with, and more accurate results may be anticipated if an upwash which changes more gently with time is used. The purpose of this Note is to suggest a series of airfoil motions which give upwashes which are as smooth functions of time as one desires for application of a finite difference approximation. Also, it is shown how the harmonic aerodynamic force coefficients may be calculated from the aerodynamic loadings corresponding to these upwashes when the perturbation motion is governed by linearized aerodynamics.

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II. General Considerations

In a modal approach, the normal displacement $Z_j(x,t)$ in mode j of a point x on the airfoil at time t in an arbitrary oscillation may be written as

$$Z_j(x,t) = cf_j(x)q_j(t) \tag{1}$$

where c is the airfoil chord, $f_j(x)$ is the jth modal function, and $q_j(t)$ is the jth generalized coordinate. If the oscillating airfoil is immersed in an airstream of density ρ and of speed V in the direction of the x axis, the airfoil experiences a loading distribution $L_j(x,t)$ (pressure force per unit area) which may be written as

$$L_j(x,t) = \rho V^2 \ell_j(x,t) \tag{2}$$

For application of Lagrange's equations of motion to an airfoil motion, the generalized air force $K_{jk}(t)$ defined by

$$K_{jk}(t) = \frac{1}{c}\int_0^c \ell_j(x,t) f_k(x)\,\mathrm{d}x \tag{3}$$

is required.

If

$$q_j(t) = \delta(t) \tag{4}$$

where $\delta(t)$ is Dirac's delta function, then we write

$$K_{jk}(t) = Q_{jk}(t) \tag{5}$$

where $Q_{jk}(t)$ is the particular form of $K_{jk}(t)$ for this case.

If

$$q_j(t) = e^{i\omega t} \tag{6}$$

then we write

$$K_{jk}(t) = \bar{Q}_{jk}(\omega) e^{i\omega t} \tag{7}$$

The harmonic generalized air force coefficients $\bar{Q}_{jk}(\omega)$ are required for aeroelastic analysis. We can show that the function $\bar{Q}_{jk}(\omega)$ is the Fourier transform of $Q_{jk}(t)$ by making use of the formula

$$e^{i\omega t} = \int_{-\infty}^{\infty} e^{i\omega t_0}\delta(t-t_0)\,dt_0 \tag{8}$$

and the principle of superposition to get

$$\bar{Q}_{jk}(\omega)e^{i\omega t} = \int_{-\infty}^{\infty} e^{i\omega t_0} Q_{jk}(t-t_0)\,dt_0$$

$$= e^{i\omega t}\int_{-\infty}^{\infty} e^{-i\omega t_0} Q_{jk}(t_0)\,dt_0 \tag{9}$$

and therefore,

$$\bar{Q}_{jk}(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t_0} Q_{jk}(t_0)\,dt_0 \tag{10}$$

Now, for the arbitrary generalized coordinate $q_j(t)$, we may write

$$q_j(t) = \int_{-\infty}^{\infty} q_j(t_0)\delta(t-t_0)\,dt_0 \tag{11}$$

Again, on using the principle of superposition, we get from Eqs. (5) and (11), for the arbitrary generalized coordinate $q_j(t)$,

$$K_{jk}(t) = \int_{-\infty}^{\infty} q_j(t_0)Q_{jk}(t-t_0)\,dt_0 \tag{12}$$

On taking the Fourier transform of Eq. (12), we get

$$\bar{K}_{jk}(\omega) = \bar{q}_j(\omega)\bar{Q}_{jk}(\omega) \tag{13}$$

where $\bar{K}_{jk}(\omega)$ and $\bar{q}_j(\omega)$ are Fourier transforms obtained, respectively, from $K_{jk}(t)$ and $q_j(t)$ in the same manner as $\bar{Q}_{jk}(\omega)$ is obtained in Eq. (10) from $Q_{jk}(t)$. From Eq. (13), we get, finally,

$$\bar{Q}_{jk}(\omega) = \bar{K}_{jk}(\omega)/\bar{q}_j(\omega) \tag{14}$$

Provided that $\bar{q}_j(\omega)\neq 0$, we can obtain the harmonic generalized air force coefficient $\bar{Q}_{jk}(\omega)$ directly from Eq. (14). If the function $q_j(t)$ of Eq. (1) is given analytically, then $\bar{q}_j(\omega)$ may be determined analytically. In transonic flow, the function $K_{jk}(t)$ may be determined only numerically, so numerical integration has to be used in determining its Fourier transform $\bar{K}_{jk}(\omega)$. The function $q_j(t)$ needs to be such that the corresponding $K_{jk}(t)$ may be determined accurately and be of such a form that the determination of $\bar{K}_{jk}(\omega)$ is feasible.

III. Choice of Generalized Coordinate $q_j(t)$

We take the function $q_j(t)$ and its first $(n-2)$ derivatives to be continuous for all t and to be zero outside the range $(0,T)$ of t. The range $(0, T)$ is divided into n equal subranges by the points $t_r = rT/n$, $r=0(1)n$. If in each subrange we take $q_j(t)$ to be a polynomial of degree $(n-1)$ in t and impose the condition

$$\int_{-\infty}^{\infty} q_j(t)\,dt = 1 \tag{15}$$

we find that

$$q_j(t) = \frac{(n/T)^n}{(n-1)!}\sum_{r=0}^{n}(-1)^r\binom{n}{r}\left((t-t_r)^{n-1}H(t-t_r)\right) \tag{16}$$

The Fourier transform $\bar{q}_j(\omega)$ turns out to be

$$\bar{q}_j(\omega) = \left(\frac{1-e^{-i\omega T/n}}{i\omega T/n}\right)^n \tag{17}$$

This transform $\bar{q}_j(\omega)$ has zeros at $\omega = \pm 2np\pi/T$, $p=1,2,\ldots$. Therefore, if we wish to evaluate $\bar{Q}_{jk}(\omega)$ from Eq. (14) for $0\leq\omega<\Omega$, we must take $T<2n\pi/\Omega$. Note that the higher the value of Ω, the lower the value of T must be. We note that as $T\rightarrow 0$, the function $q_j(t)$ tends weakly to the Dirac delta function $\delta(t)$ irrespective of the value of n, and the Fourier transform $\bar{q}_j(\omega)=1$ is obtained. Therefore, according to Eq. (14), $\bar{K}_{jk}(\omega)$ tends to $\bar{Q}_{jk}(\omega)$.

Corresponding to $q_j(t)$ being zero outside the range $(0, T)$ of t, the generalized air force $K_{jk}(t)$ is zero for $t<0$ and tends to zero as $t\rightarrow\infty$. It is thus possible to obtain the Fourier transform $\bar{K}_{jk}(\omega)$ would be known only up to some finite value of t, and the integral would have to be truncated.

Instead of the function $q_j(t)$ defined above, which tends to the Dirac delta function as $T\rightarrow 0$, we could take the function $q_j^{(I)}(t)$ defined by

$$q_j^{(I)}(t) = \int_{-\infty}^{t} q_j(t_0)\,dt_0 \tag{18}$$

which tends to the Heaviside unit function $H(t)$ as $T\rightarrow 0$. The choice of $q_j^{(I)}(t)$ rather than $q_j(t)$ would seem to be in harmony with the normal practice in the literature of using a step change of variable rather than an impulsive change. However, we indicate below that the process of obtaining the Fourier transform of the corresponding generalized force $K_{jk}^{(I)}(t)$, for use in Eq. (14), is not as direct as it is for $K_{jk}(t)$, and the accuracy is not as good. For these reasons, the authors' preferred procedure for obtaining $\bar{Q}_{jk}(\omega)$ is the one using $q_j(t)$ rather than $q_j^{(I)}(t)$.

The function $q_j^{(I)}(t)$ is zero to $t<0$ and equal to unity for $t>T$ [see Eq. (15)]. The corresponding generalized air force $K_{jk}^{(I)}(t)$ is zero for $t<0$, but, in general, tends to a nonzero finite value as $t\rightarrow\infty$. To get the Fourier transform, we write

$$K_{jk}^{(I)}(t) = K_{jk}^{(I)}(+\infty)H(t) + J_{jk}(t) \tag{19}$$

The function $J_{jk}(t)$ is zero for $t<0$ and tends to zero as $t\rightarrow\infty$. Again, it is possible to obtain the Fourier transform $\bar{J}_{jk}(\omega)$ by numerical means directly from the integral defining it, but because it tends to zero rather more slowly than does $K_{jk}(t)$ as $t\rightarrow\infty$ and truncation must be used, the accuracy is impaired compared with that of $\bar{K}_{jk}(\omega)$. On taking the Fourier transform of Eq. (19), we get

$$\bar{K}_{jk}^{(I)}(\omega) = \frac{1}{i\omega}K_{jk}^{(I)}(+\infty) + \bar{J}_{jk}(\omega) \tag{20}$$

The Fourier transform $\bar{q}_j^{(I)}(\omega)$ of $q_j^{(I)}(t)$ is

$$\bar{q}_j^{(I)}(\omega) = \frac{1}{i\omega}\left(\frac{1-e^{-i\omega T/n}}{i\omega T/n}\right)^n \tag{21}$$

The value of $\bar{Q}_{jk}(\omega)$ may again be obtained from Eq. (14) for $0\leq\omega<\Omega$ provided that $T<2n\pi/\Omega$, and there is no problem with the infinities of $\bar{q}_j^{(I)}(\omega)$ and $\bar{K}_{jk}^{(I)}(\omega)$ at $\omega=0$. As $T\rightarrow 0$, the Fourier transform $\bar{q}_j^{(I)}(\omega)=1/i\omega$ is obtained; thus in this case $\bar{K}_{jk}^{(I)}(\omega)$ does not tend to $\bar{Q}_{jk}(\omega)$.

The value of n to be chosen in the definition of the function $q_j(t)$ should be such that the resulting function $q_j(t)$ is adequately smooth for a finite difference scheme to be able to cope with the upwash $W_j(x,t)$ given by

$$W_j(x,t) = V\frac{\partial Z_j}{\partial x}(x,t) + \frac{\partial Z_j}{\partial t}(x,t)$$

$$= V\left\{c\frac{\mathrm{d}}{\mathrm{d}x}f_j(x)q_j(t) + \frac{c}{V}f_j(x)\frac{\mathrm{d}q_j(t)}{\mathrm{d}t}\right\} \tag{22}$$

The time interval T must be so chosen that $T<2n\pi/\Omega$ and the time step for the finite difference approximation be chosen correspondingly to be small enough for adequate accuracy to be obtained. The number of time steps required before the generalized air force $K_{jk}(t)$ is effectively zero will have to be determined by experience, but it will depend on n, T, and the time step.

IV. Conclusions

A suitable form of the generalized coordinate $q_j(t)$, defining the modal motion of an airfoil, has been prescribed as a transient in which there are no changes severe enough to mar the accuracy of finite difference approximations to governing aerodynamic equations. The form of $q_j(t)$ is an approximation to the Dirac delta function. From the resulting generalized air forces $K_{jk}(t)$, it is possible to determine the harmonic air force coefficients $\bar{Q}_{jk}(\omega)$ that are required for aeroelastic analysis.

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Asymptotic Features of Shock-Wave Boundary-Layer Interaction

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Nomenclature

C = Chapman-Rubesin constant, $[(T_0+198.6)/(T_w+198.6)](T_w/T_0)^{1/2}$
C_f = $[\mu(\partial u/\partial y)]_{y=0}/\frac{1}{2}\rho_0 U_0^2$
M_0 = freestream Mach number
p = pressure (dimensional)
p_b = minimum surface pressure that occurs immediately ahead of interaction region
p_0 = freestream pressure (undisturbed)
P = $(M_0^2-1)^{1/4}[p(x_s)-p_0]/(R^{1/4}\gamma C^{1/4}M_0^2 p_0)$
P' = $[R^{1/8}(T_w/T_0)^{3/2}C^{1/8}]/[\gamma M_0^2(M_0^2-1)^{1/8}\lambda^{7/4}](\mathrm{d}p/\mathrm{d}x)|_{x=x_s}$
R = Reynolds number, $U_0 x_s/\nu_0$
T_w = temperature at wall
T_0 = temperature in undisturbed freestream
U_0 = undisturbed freestream velocity
U_1 = velocity behind incident shock
x_s = distance of separation point from leading edge
X = $[(x-x_s)/x_s](K/C)^{3/8}(M_0-1)^{3/8}\lambda^{5/4}(T_0/T_w)^{3/2}$
λ = 0.3321

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Introduction

IT is well known that when a sufficiently strong shock wave strikes a laminar boundary layer, the boundary layer separates ahead of the point of shock incidence, and the flow features of the separation region are independent of the mode of inducing separation and the direct influences of downstream geometry. Chapman et al.[1] were the first to give a simple physical explanation of this free-interaction phenomenon. A self-consistent mathematical theory was developed by Lighthill[2] for interaction of a weak shock wave with a boundary layer. The generalization of this linear theory to include nonlinear disturbances on the boundary-layer scale led to the triple-deck formulation of Stewartson and Williams[3] and Neiland.[4] A comprehensive review of this asymptotic large Reynolds number theory is given by Stewartson.[5] Since solutions based on the triple-deck theory can be reliably evaluated, it is of interest to determine the Reynolds number range within which they are accurate so that they may be used to check the accuracy of more complex solution procedures.

The purpose of the present Note is to apply the semi-implicit method of MacCormack[6] to solve the Navier-Stokes equations numerically and to evaluate certain features of the free-interaction phenomenon that occurs when a shock wave impinges on a Blasius boundary layer. Comparisons are made with predictions of the triple-deck theory and experiment. The results include pressure and skin-friction distributions in the free-interaction region for various values of Reynolds number. The functional dependence on Reynolds number of the shock strength required for incipient separation is determined. The triple-deck (asymptotic) theory is viewed in the light of these results, and conclusions are drawn with regard to this theory being used as a test for numerical schemes.

Results and Discussion

A. Incipient Separation

Incipient separation is that condition in which the wall shear is positive everywhere except at one point where it vanishes. Estimates of the scaling of the magnitude of the required shock strength and the streamwise extent of its interaction with the boundary layer have been given by Neiland[4] and Sychev.[7] According to these estimates, the pressure scales as $R^{-1/4}$ and the streamwise extent of interaction scales as $R^{-3/8}$.

To determine incipient separation by computation for a given Mach number and Reynolds number, two angles of shock incidence are guessed, such that for one of them the flow remains attached and for the other it separates. The interval between these two angles is divided into sub-intervals of, for example, 0.1, and the flowfield is computed at these shock incidences. The results then give two relatively close angles. The boundary layer separates for one angle, but not for the other. The interval between these two angles can be further subdivided and so on until the desired accuracy is obtained. The numerical results obtained bear out the preceding estimate of dependence on Reynolds number. In Fig. 1, the scaled shock strength for incipient separation determined from the present calculations is plotted vs Reynolds number. Straight horizontal lines have been drawn through the computed points. The deviation of the computed points from the straight line predicted by asymptotic theory